Binary relations R operating on a set A are said to be asymmetric, antisymmetric or irreflexive according to the following definitions for arbitrary but fixed R and A.

$$\begin{align} \textit{asymmetric}\,(R, A) &\iff \forall\, x, y \in A : R(x, y) \Rightarrow \neg R(y, x) \tag{1} \\ \textit{antisymmetric}\,(R, A) &\iff \forall\, x, y \in A : (R(x, y) \wedge R(y, x)) \Rightarrow (x = y) \tag{2} \\ \textit{irreflexive}\,(R, A) &\iff \forall\, x \in A : \neg R(x, x) \tag{3} \end{align}$$

We show that, again for arbitrary but fixed R and A, the implication

$$\textit{asymmetric}\,(R, A) \Rightarrow (\textit{antisymmetric}\,(R, A) \wedge \textit{irreflexive}\,(R, A))$$

holds by assuming that $\textit{asymmetric}\,(R, A)$ is true and showing that $\textit{antisymmetric}\,(R, A)$ and $\textit{irreflexive}\,(R, A)$ then hold.

It is elementary that, for arbitrary but fixed $x, y \in A$ the statement $R(x, y) \wedge R(y, x)$ in (2) is a contradiction. $R(x, y)$ implies that $\neg R(y, x)$, therefore $R(x, y) \wedge R(y, x)$ can never be true for fixed x, y under the assumption of (1). Since the left-hand side of the implication in (2) is always false the statement will always be true. We have thus shown that $\textit{antisymmetric}\,(R, A)$ holds.

If $R(x, x)$ were true for arbitrary but fixed $x \in A$ it would lead to $R(x, x) \Rightarrow \neg R(x, x)$ being false. Since that is a contradiction (we know that (1) holds for our R and A) we conclude that $\neg R(x, x)$. We have thus shown that $\textit{irreflexive}\,(R, A)$ holds.