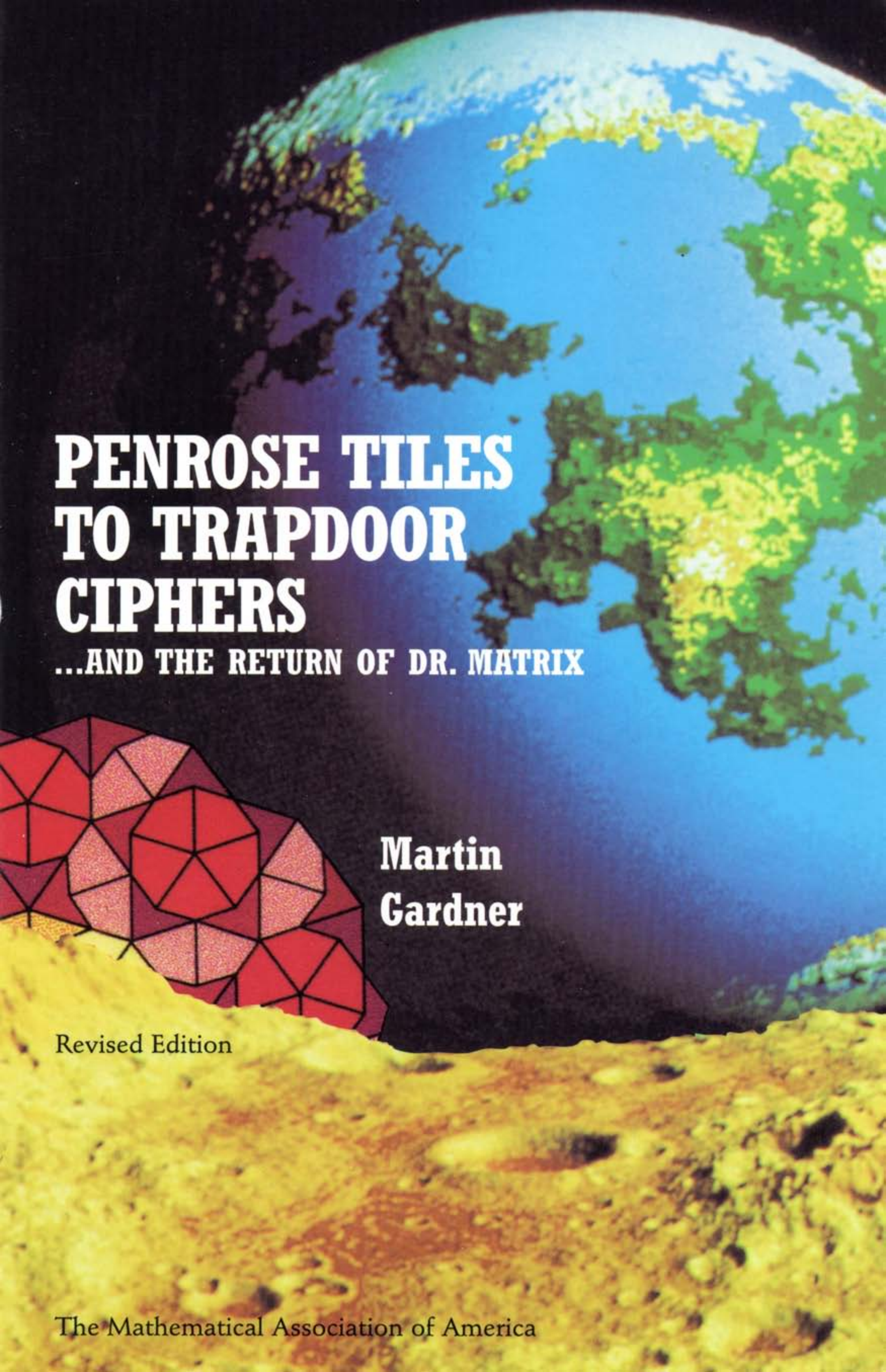


PENROSE TILES TO TRAPDOOR CIPHERS

...AND THE RETURN OF DR. MATRIX

Martin Gardner

Revised Edition

The Mathematical Association of America

1

The Wonders of a Planiverse

"Planiversal scientists are not a very common breed."

—Alexander Keewatin Dewdney

As far as anyone knows the only existing universe is the one we live in, with its three dimensions of space and one of time. It is not hard to imagine, as many science-fiction writers have, that intelligent organisms could live in a four-dimensional space, but two dimensions offer such limited degrees of freedom that it has long been assumed intelligent two-space life forms could not exist. Two notable attempts have nonetheless been made to describe such organisms.

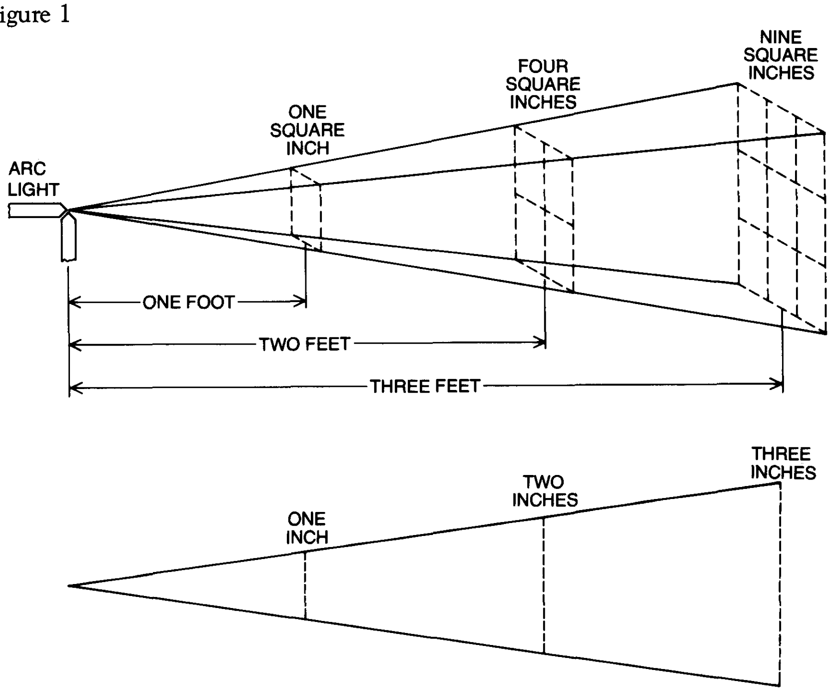
In 1884 Edwin Abbott Abbott, a London clergyman, published his satirical novel *Flatland.* Unfortunately the book leaves the reader almost entirely in the dark about Flatland's physical laws and the technology developed by its inhabitants, but the situation was greatly improved in 1907 when Charles Howard Hinton published *An Episode of Flatland.* Although written in a flat style and with cardboard characters, Hinton's story provided the first glimpses of the possible science and technology of the two-dimensional world. His eccentric book is, alas, long out of print, but you can read about it in the chapter "Flatlands" in my book *The Unexpected Hanging and Other Mathematical Diversions* (Simon & Schuster, 1969).

In "Flatlands" I wrote: "It is amusing to speculate on two-dimensional physics and the kinds of simple mechanical devices that would be feasible in a flat world." This remark caught the attention of Alexander Keewatin Dewdney, a computer scientist at the University of Western Ontario. Some of his early speculations on the subject were set down in 1978 in a university report and in 1979 in "Exploring the Planiverse," an article in *Journal of Recreational Mathematics* (Vol. 12, No. 1, pages 16–20; September). Later in 1979 Dewdney also privately published "Two-dimensional Science and Technology," a 97-page tour de force. It is hard to believe, but Dewdney actually lays the groundwork for what he calls a planiverse: a possible two-dimensional world. Complete with its own laws of chemistry, physics, astronomy, and biology, the planiverse is closely analogous to our own universe (which he calls the steriverse) and is apparently free of contradictions. I should add that this remarkable achievement is an amusing hobby for a mathematician whose serious contributions have appeared in some 30 papers in technical journals.

Dewdney's planiverse resembles Hinton's in having an earth that he calls (as Hinton did) Astria. Astria is a disklike planet that rotates in planar space. The Astrians, walking upright on the rim of the planet, can distinguish east and west and up and down. Naturally there is no north or south. The "axis" of Astria is a point at the center of the circular planet. You can think of such a flat planet as being truly two-dimensional or you can give it a very slight thickness and imagine it as moving between two frictionless planes.

As in our world, gravity in a planiverse is a force between objects that varies directly with the product of their masses, but it varies inversely with the *linear* distance between them, not with the square of that distance. On the assumption that forces such as light and gravity in a planiverse move in straight lines, it is easy to see that the intensity of such forces must vary inversely with linear distance. The familiar textbook figure demonstrating that in our world the intensity of light varies inversely with the square of distance is shown at the top of Figure 1. The obvious planar analogue is shown at the bottom of the illustration.

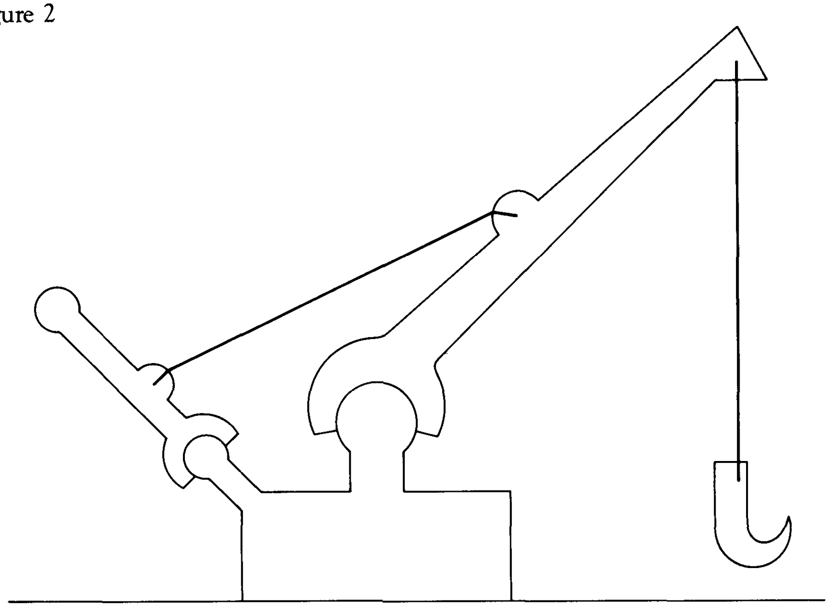
Figure 1



To keep his whimsical project from "degenerating into idle speculation" Dewdney adopts two basic principles. The "principle of similarity" states that the planiverse must be as much like the steriverse as possible: a motion not influenced by outside forces follows a straight line, the flat analogue of a sphere is a circle, and so on. The "principle of modification" states that in those cases where one is forced to choose between conflicting hypotheses, each one equally similar to a steriversal theory, the more fundamental one must be chosen and the other modified. To determine which hypothesis is more fundamental Dewdney relies on the hierarchy in which physics is more fundamental than chemistry, chemistry more fundamental than biology, and so on.

To illustrate the interplay between levels of theory Dewdney considers the evolution of the planiversal hoist in Figure 2. The engineer who designed it first gave it arms thinner than those in the illustration, but when a metallurgist pointed out that planar materials fracture more easily than their three-space counterparts, the engineer made the arms

Figure 2



thicker. Later a theoretical chemist, invoking the principles of similarity and modification at a deeper level, calculated that the planiversal molecular forces are much stronger than had been suspected, and so the engineer went back to thinner arms.

The principle of similarity leads Dewdney to posit that the planiverse is a three-dimensional continuum of space–time containing matter composed of molecules, atoms, and fundamental particles. Energy is propagated by waves, and it is quantized. Light exists in all its wavelengths and is refracted by planar lenses, making possible planiversal eyes, planiversal telescopes, and planiversal microscopes. The planiverse shares with the steriverse such basic precepts as causality; the first and second laws of thermodynamics; and laws concerning inertia, work, friction, magnetism, and elasticity.

Dewdney assumes that his planiverse began with a big bang and is currently expanding. An elementary calculation based on the inverse-linear gravity law shows that regardless of the amount of mass in the planiverse the expansion must eventually halt, so that a contracting phase will begin. The Astrian night sky will of course be a semicircle along which are scattered twinkling points of light. If the stars have proper motions, they will continually be occulting one another. If Astria has a sister planet, it will over a period of time occult every star in the sky.

We can assume that Astria revolves around a sun and rotates, thereby creating day and night. In a planiverse, Dewdney discovered, the only stable orbit that continually retraces the same path is a perfect circle. Other stable orbits roughly elliptical in shape are possible, but the axis of the ellipse rotates in such a way that the orbit never exactly closes. Whether planiversal gravity would allow a moon to have a stable orbit around Astria remains to be determined. The difficulty is due to the sun's gravity, and resolving the question calls for work on the planar analogue of what our astronomers know as the three-body problem.

Dewdney analyzes in detail the nature of Astrian weather, using analogies to our seasons, winds, clouds, and rain. An Astrian river would be indistinguishable from a lake except that it might have faster currents. One peculiar feature of Astrian geology is that water cannot flow around a rock as it does on the earth. As a result rainwater steadily accumulates behind any rock on a slope, tending to push the rock downhill: the gentler the slope is, the more water accumulates and the stronger the push is. Dewdney concludes that given periodic rainfall the Astrian surface would be unusually flat and uniform. Another consequence of the inability of water to move sideways on Astria is that it would become trapped in pockets within the soil, tending to create large areas of treacherous quicksand in the hollows of the planet. One hopes, Dewdney writes, that rainfall is infrequent on Astria. Wind too would have much severer effects on Astria than on the earth because like rain it cannot "go around" objects.

Dewdney devotes many pages to constructing a plausible chemistry for his planiverse, modeling it as much as possible on three-dimensional matter and the laws of quantum mechanics. Figure 3 shows Dewdney's periodic table for the first 16 planiversal elements. Because the first two are so much like their counterparts in our world, they are called hydrogen and helium. The next 10 have composite names to suggest the steriversal elements they most resemble; for example, lithrogen combines the properties of lithium and nitrogen. The next four are named after Hinton, Abbott, and the young lovers in Hinton's novel, Harold Wall and Laura Cartwright.

In the flat world atoms combine naturally to form molecules, but of course only bonding that can be diagrammed by a planar graph is allowed. (This result follows by analogy from the fact that intersecting bonds do not exist in steriversal chemistry.) As in our world, two asymmetric molecules can be mirror images of each other, so that neither

ATOMIC NUMBER	NAME	SYMBOL	1s	2s	2p	3s	3p	3d	4s	4p . . .	VALENCE
			SHELL STRUCTURE								
1	HYDROGEN	H	1								1
2	HELIUM	He	2								2
3	LITROGEN	Lt	2	1							1
4	BEROXYGEN	Bx	2	2							2
5	FLUORON	Fl	2	2	1						3
6	NEOCARBON	Nc	2	2	2						4
7	SODALINUM	Sa	2	2	2	1					1
8	MAGNILICON	Mc	2	2	2	2					2
9	ALUPHORUS	Ap	2	2	2	2	1				3
10	SULFICON	Sp	2	2	2	2	2				4
11	CHLOPHORUS	Cp	2	2	2	2	2	1			5
12	ARGOFUR	Af	2	2	2	2	2	2			6
13	HINTONIUM	Hn	2	2	2	2	2	2	1		1
14	ABBOGEN	Ab	2	2	2	2	2	2	2		2
15	HAROLDIUM	Wa	2	2	2	2	2	2	2	1	3
16	LAURANIUM	La	2	2	2	2	2	2	2	2	4

Figure 3

one can be "turned over" to become identical with the other. There are striking parallels between planiversal chemistry and the behavior of steriversal monolayers on crystal surfaces [see "Two-dimensional Matter," by J. G. Dash; *Scientific American,* May 1973]. In our world molecules can form 230 distinct crystallographic groups, but in the planiverse they can form only 17. I am obliged to pass over Dewdney's speculations about the diffusion of molecules, electrical and magnetic laws, analogues of Maxwell's equations, and other subjects too technical to summarize here.

Dewdney assumes that animals on Astria are composed of cells that cluster to form bones, muscles, and connective tissues similar to those found in steriversal biology. He has little difficulty showing how these bones and muscles can be structured to move appendages in such a way that the animals can crawl, walk, fly, and swim. Indeed, some of these movements are easier in a planiverse than in our world. For example, a steriversal animal with two legs has considerable difficulty balancing while walking, whereas in the planiverse if an animal has both legs on the ground, there is no way it can fall over. Moreover, a flying planiversal animal cannot have wings and does not need them to fly; if the body of the animal is aerodynamically shaped, it can act as a wing (since air can go around it only in the plane). The flying animal could be propelled by a flapping tail.

Calculations also show that Astrian animals probably have much lower metabolic rates than terrestrial animals because relatively little heat is lost through the perimeter of their body. Furthermore, animal bones can be thinner on Astria than they are on the earth, because they have less weight to support. Of course, no Astrian animal can have an open tube extending from its mouth to its anus, because if it did, it would be cut in two.

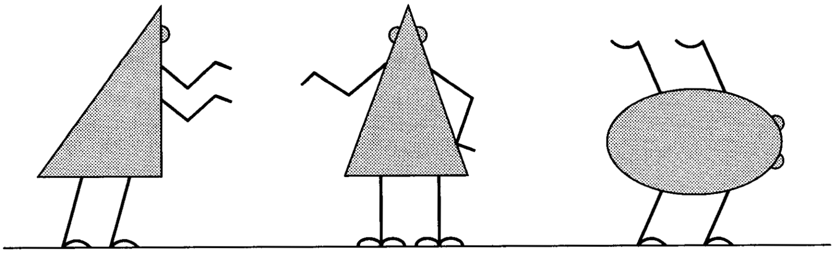
In the appendix to his book *The Structure and Evolution of the Universe* (Harper, 1959) G. J. Whitrow argues that intelligence could not evolve in two-space because of the severe restrictions two dimensions impose on nerve connections. "In three or more dimensions," he writes, "any number of [nerve] cells can be connected with [one another] in pairs without intersection of the joins, but in two dimensions the maximum number of cells for which this is possible is only four." Dewdney easily demolishes this argument, pointing out that if nerve cells are allowed to fire nerve impulses through "crossover points," they can form flat networks as complex as any in the steriverse. Planiversal minds would operate more slowly than steriversal ones, however, because in the two-dimensional networks the pulses would encounter more interruptions. (There are comparable results in the theory of two-dimensional automatons.)

Dewdney sketches in detail the anatomy of an Astrian female fish with a sac of unfertilized eggs between its two tail muscles. The fish has an external skeleton, and nourishment is provided by the internal circulation of food vesicles. If a cell is isolated, food enters it through a membrane that can have only one opening at a time. If the cell is in contact with other cells, as in a tissue, it can have more than one opening at a time because the surrounding cells are able to keep it intact. We can of course see every internal organ of the fish or of any other planiversal life form, just as a four-dimensional animal could see all our internal organs.

Dewdney follows Hinton in depicting his Astrian people schematically, as triangles with two arms and two legs. Hinton's Astrians, however, always face in the same direction: males to the east and females to the west. In both sexes the arms are on the front side, and there is a single eye at the top of the triangle, as shown in Figure 4. Dewdney's Astrians are bilaterally symmetrical, with an arm, a leg, and an eye on each side, as shown in the illustration's center. Hence these Astrians, like terrestrial birds or horses, can see in opposite directions. Naturally the only way for one Astrian to pass another is to crawl or leap over him. My conception of an Astrian bug-eyed monster is shown at the right in the illustration. This creature's appendages serve as either arms or legs, depending on which way it is facing, and its two eyes provide binocular vision. With only one eye an Astrian would have a largely one-dimensional visual world, giving him a rather narrow perception of reality. On the other hand, parts of objects in the planiverse might be distinguished by their color, and an illusion of depth might be created by the focusing of the lens of the eye.

On Astria building a house or mowing a lawn requires less work

Figure 4

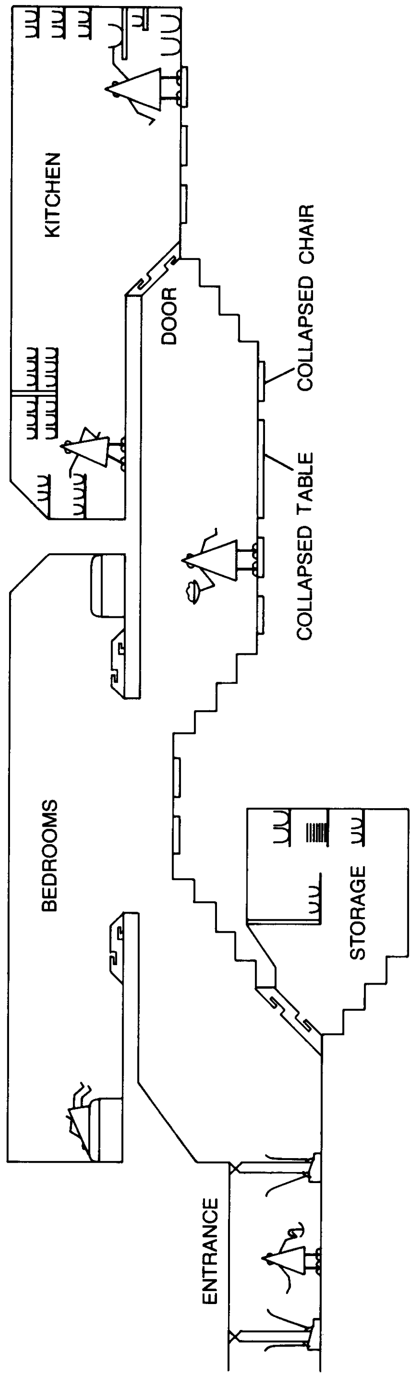


than it does on the earth because the amount of material involved is considerably smaller. As Dewdney points out, however, there are still formidable problems to be dealt with in a two-dimensional world: "Assuming that the surface of the planet is absolutely essential to support life-giving plants and animals, it is clear that very little of the Astrian surface can be disturbed without inviting the biological destruction of the planet. For example, here on earth we may build a modest highway through the middle of several acres of rich farmland and destroy no more than a small percentage of it. A corresponding highway on Astria with destroy *all* the 'acreage' it passes over. . . . Similarly, extensive cities would quickly use up the Astrian countryside. It would seem that the only alternative for the Astrian technological society is to go underground." A typical subterranean house with a living room, two bedrooms, and a storage room is shown in Figure 5. Collapsible chairs and tables are stored in recesses in the floors to make the rooms easier to walk through.

The many simple three-dimensional mechanical elements that have obvious analogues on Astria include rods, levers, inclined planes, springs, hinges, ropes, and cables (see Figure 6, top). Wheels can be rolled along the ground, but there is no way to turn them on a fixed axle. Screws are impossible. Ropes cannot be knotted; but by the same token, they never tangle. Tubes and pipes must have partitions, to keep their sides in place, and the partitions have to be opened (but never all of them at once) to allow anything to pass through. It is remarkable that in spite of these severe constraints many flat mechanical devices can be built that will work. A faucet designed by Dewdney is shown in Figure 6, bottom. To operate it the handle is lifted. This action pulls the valve away from the wall of the spout, allowing the water to flow out. When the handle is released, the spring pushes the valve back.

The device shown in Figure 7 serves to open and close a door (or a wall). Pulling down the lever at the right forces the wedge at the

Figure 5


KITCHEN
DOOR
COLLAPSED CHAIR
COLLAPSED TABLE
BEDROOMS
STORAGE
ENTRANCE

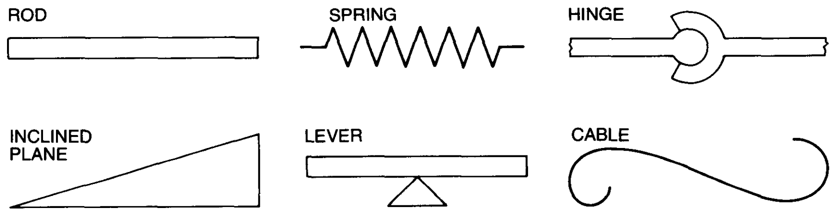


Figure 6 (top)

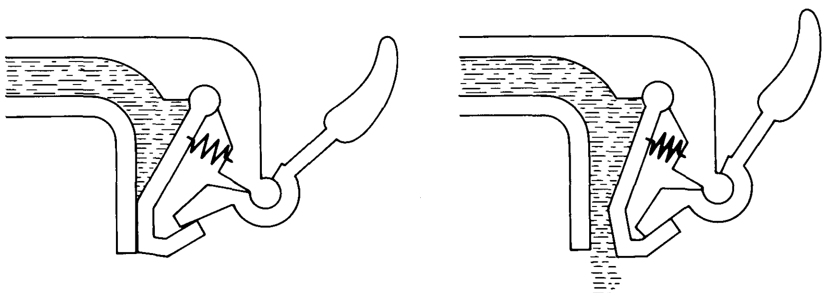


Figure 6 (bottom)

bottom to the left, thereby allowing the door to swing upward (carrying the wedge and the levers with it) on a hinge at the top. The door is opened from the left by pushing up on the other lever. The door can be lowered from either side and the wedge moved back to stabilize the wall by moving a lever in the appropriate direction. This device and the faucet are both mechanisms with permanent planiversal hinges: circular knobs that rotate inside hollows but cannot be removed from them.

Figure 8 depicts a planiversal steam engine whose operation parallels that of a steriversal engine. Steam under pressure is admitted into the cylinder of the engine through a sliding valve that forms one of its walls (*top*). The steam pressure causes a piston to move to the right until steam can escape into a reservoir chamber above it. The subsequent loss of pressure allows the compound leaf spring at the right of the cylinder to drive the piston back to the left *(bottom)*. The sliding valve is

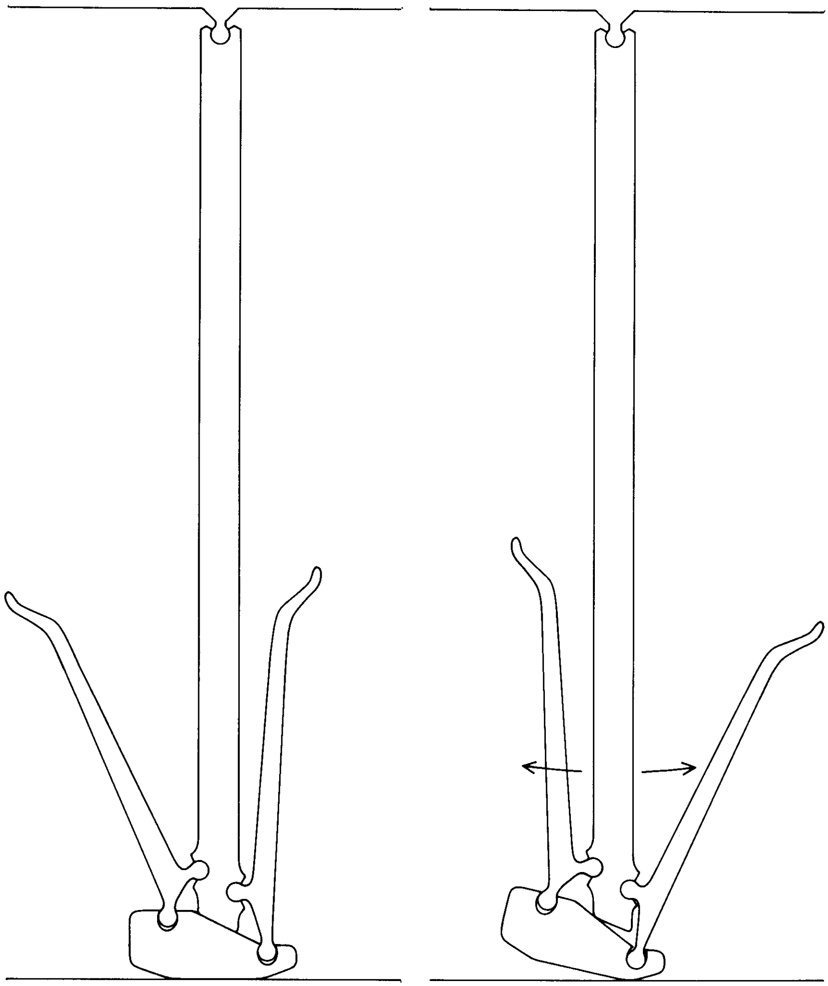


Figure 7

closed as the steam escapes into the reservoir, but as the piston moves back it reopens, pulled to the right by a spring-loaded arm.

Figure 9 depicts Dewdney's ingenious mechanism for unlocking a door with a key. This planiversal lock consists of three slotted tumblers (*a*) that line up when a key is inserted (*b*) so that their lower halves move as a unit when the key is pushed (*c*). The pushing of the key is transmitted through a lever arm to the master latch, which pushes

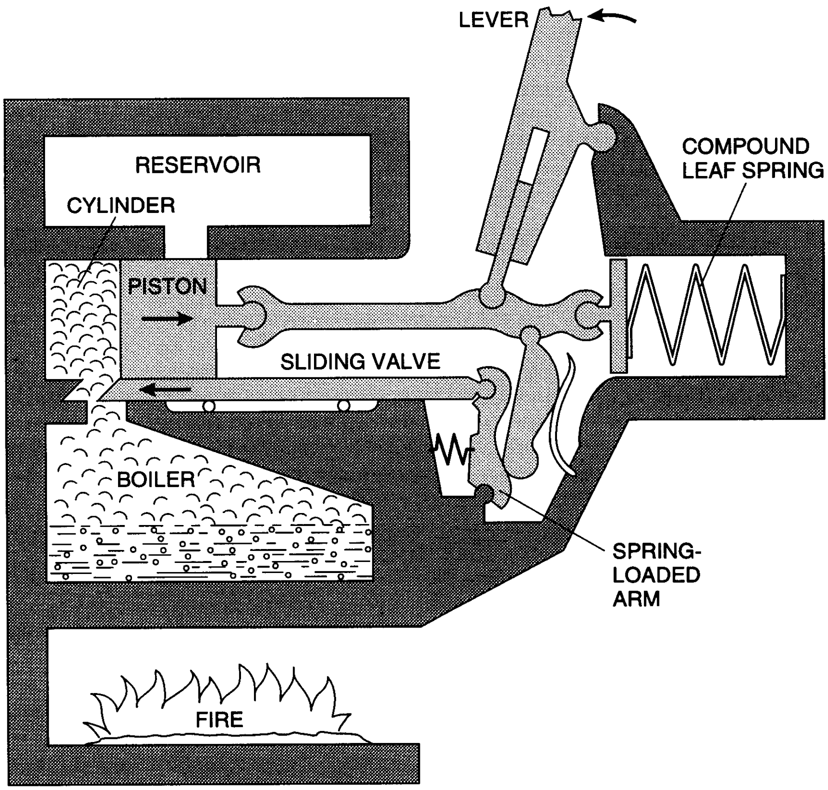


Figure 8

down on a slave latch until the door is free to swing to the right (*d*). The bar on the lever arm and the lip on the slave latch make the lock difficult to pick. Simple and compound leaf springs serve to return all the parts of the lock except the lever arm to their original positions when the door is opened and the key is removed. When the door closes, it strikes the bar on the lever arm, thereby returning that piece to its original position as well. This flat lock could actually be employed in the steriverse; one simply inserts a key without twisting it.

"It is amusing to think," writes Dewdney, "that the rather exotic design pressures created by the planiversal environment could cause us to think about mechanisms in such a different way that entirely novel solutions to old problems arise. The resulting designs, if steriversally practical, are invariably space-saving."

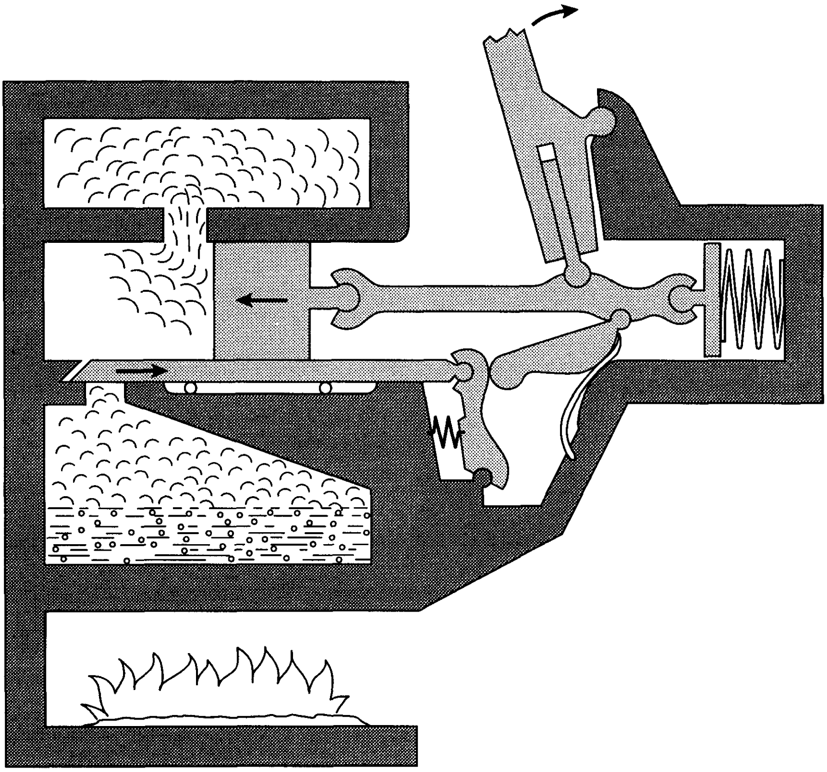


Figure 8 (continued)

Thousands of challenging planiversal problems remain unsolved. Is there a way, Dewdney wonders, to design a two-dimensional windup motor with flat springs or rubber bands that would store energy? What is the most efficient design for a planiversal clock, telephone, book, typewriter, car, elevator or computer? Will some machines need a substitute for the wheel and axle? Will some need electric power?

There is a curious pleasure in trying to invent machines for what Dewdney calls "a universe both similar to and yet strangely different from ours." As he puts it, "from a small number of assumptions many phenomena seem to unfurl, giving one the sense of a kind of separate existence of this two-dimensional world. One finds oneself speaking, willy-nilly, of *the* planiverse as opposed to *a* planiverse. . . . [For] those

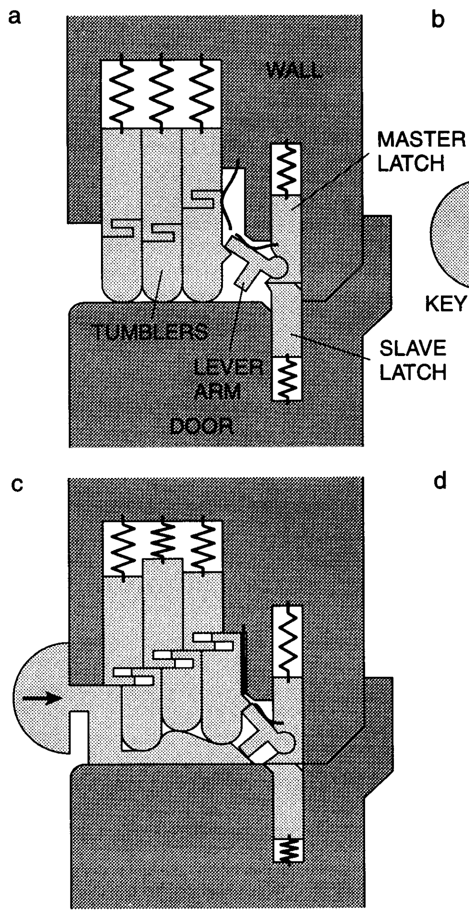


Figure 9

who engage in it positively, there is a kind of strange enjoyment, like [that of] an explorer who enters a land where his own perceptions play a major role in the landscape that greets his eyes."

Some philosophical aspects of this exploration are not trivial. In constructing a planiverse one sees immediately that it cannot be built without a host of axioms that Leibniz called the "compossible" elements of any possible world, elements that allow a logically consistent structure. Yet as Dewdney points out, science in our universe is based mainly on observations and experiments, and it is not easy to find any underlying axioms. In constructing a planiverse we have nothing to observe. We can only perform *gedanken* experiments (thought experiments) about what might be observed. "The experimentalist's loss," observes Dewdney, "is the theoretician's gain."

A marvelous exhibit could be put on of working models of planiversal machines, cut out of cardboard or sheet metal, and displayed on a surface that slopes to simulate planiversal gravity. One can also imagine beautiful cardboard exhibits of planiversal landscapes, cities, and houses. Dewdney has opened up a new game that demands knowledge of both science and mathematics: the exploration of a vast fantasy world about which at present almost nothing is known.

It occurs to me that Astrians would be able to play two-dimensional board games but that such games would be as awkward for them as three-dimensional board games are for us. I imagine them, then, playing a variety of linear games on the analogue of our 8-by-8 chessboard. Several games of this type are shown in Figure 10. Part *a* shows the start of a checkers game. Pieces move forward only, one cell at a time, and jumps are compulsory. The linear game is equivalent to a game of regular checkers with play confined to the main diagonal of a standard board. It is easy to see how the second player wins in rational play and how in misère, or "giveaway," checkers the first player wins just as easily. Linear checkers games become progressively harder to analyze as longer boards are introduced. For example, which player wins standard linear checkers on the 11-cell board when each player starts with checkers on the first four cells at his end of the board?

Part *b* in the illustration shows an amusing Astrian analogue of chess. On a linear board a bishop is meaningless and a queen is the same as a rook, so the pieces are limited to kings, knights, and rooks. The only rule modification needed is that a knight moves two cells in either direction and can jump an intervening piece of either color. If

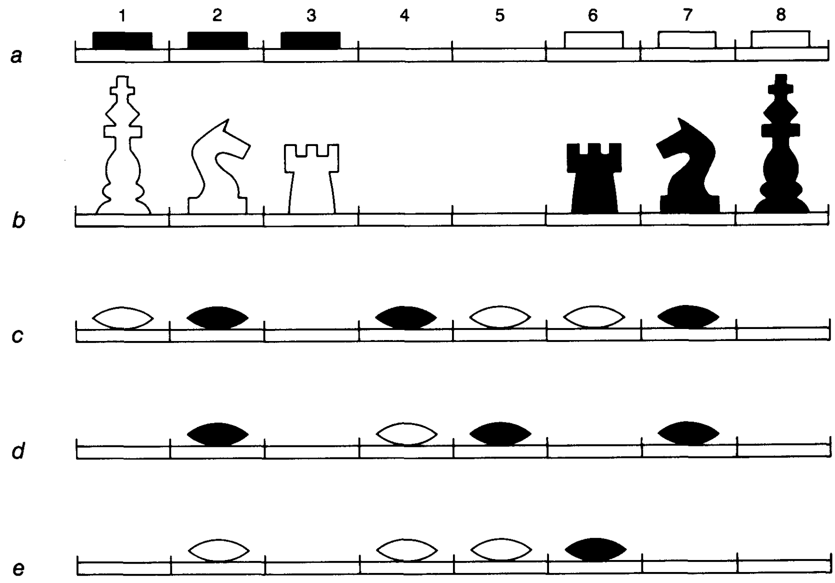


Figure 10

the game is played rationally, will either White or Black win or will the game end in a draw? The question is surprisingly tricky to answer.

Linear go, played on the same board, is by no means trivial. The version I shall describe was invented 10 years ago by James Marston Henle, a mathematician who is now at Smith College. Called pinch by Henle, it is published here for the first time.

In the game of pinch players take turns placing black and white stones on the cells of the linear board, and whenever the stones of one player surround the stones of the other, the surrounded stones are removed. For example, both sets of white stones shown in part *c* of Figure 10 are surrounded. Pinch is played according to the following two rules.

Rule 1: No stone can be placed on a cell where it is surrounded unless that move serves to surround a set of enemy stones. Hence in the situation shown in part *d* of the illustration White cannot play on cells 1, 3, or 8, but he can play on cell 6 because this move serves to surround cell 5.

Rule 2: A stone cannot be placed on a cell from which a stone was removed on the last play if the purpose of the move is to surround something. A player must wait at least one turn before making such a move. For example, in part *e* of the illustration assume that Black plays on cell 3 and removes the white stones on cells 4 and 5. White cannot play on cell 4 (to surround cell 3) for his next move, but he may do so for any later move. He can play on cell 5, however, because even though a stone was just removed from that cell, the move does not serve to surround anything. This rule is designed to decrease the number of stalemates, as is the similar rule in go.

Two-cell pinch is a trivial win for the second player. The three- and four-cell games are easy wins for the first player if he takes the center in the three-cell game and one of the two central cells in the four-cell one. The five-cell game is won by the second player and the six- and seven-cell games are won by the first player. The eight-cell game jumps to such a high level of complexity that it becomes very exciting to play. Fortunes often change rapidly, and in most situations the winning player has only one winning move.

Answers

In 11-cell linear checkers (beginning with Black on cells 1, 2, 3, and 4 and White on cells 8, 9, 10, and 11) the first two moves are forced: Black to 5 and White to 7. To avoid losing, Black then goes to 4, and White must respond by moving to 8. Black is then forced to 3 and White to 9. At this point Black loses with a move to 2 but wins with a move to 6. In the latter case White jumps to 5, and then Black jumps to 6 for an easy end-game victory.

On the eight-cell linear chessboard White can win in at most six moves. Of White's four opening moves, R×R is an instant stalemate and the shortest possible game. R-5 is a quick loss for White if Black plays R×R. Here White must respond with N-4, and then Black mates on his second move with R×N. This game is one of the two "fool's mates," or shortest possible wins. The R-4 opening allows Black to mate on his second or third move if he responds with N-5.

White's only winning opening is N-4. Here Black has three possible replies:

1. R×N.

In this case White wins in two moves with R×R.

2. R-5.

White wins with K-2. If Black plays R-6, White mates with N×R. If Black takes the knight, White takes the rook, Black moves N-5, and White mates by taking Black's knight

3. N-5.

This move delays Black's defeat the longest. In order to win White must check with N×R, forcing Black's king to 7. White moves his rook to 4. If Black plays K×N, White's king goes to 2, Black's K-7 is forced, and White's R×N wins. If Black plays N-3 (check), White moves the king to 2. Black can move only the knight. If he plays N-1, White mates with N-8. If Black plays N-5, White's N-8 forces Black's K×N, and then White mates with R×N.

The first player also has the win in eight-cell pinch (linear go) by opening on the second cell from an end, a move that also wins the six and seven-cell games. Assume that the first player plays on cell 2. His unique winning responses to his opponent's plays on 3, 4, 5, 6, 7, and 8 are respectively 5, 7, 7, 7, 5, and 6. I leave the rest of the game to the reader. It is not known whether there are other winning opening moves. James Henle, the inventor of pinch, assures me that the second player wins the nine-cell game. He has not tried to analyze boards with more than nine cells.

ADDENDUM

My column on the planiverse generated enormous interest. Dewdney received some thousand letters offering suggestions about flatland science and technology. In 1979 he privately printed *Two-Dimensional Science and Technology*, a monograph discussing these new results. Two years later he edited another monograph, *A Symposium of Two-Dimensional Science and Technology*. It contained papers by noted scientists, mathematicians, and laymen, grouped under the categories of physics, chemistry, astronomy, biology, and technology. *Newsweek* covered these monographs in a two-page article, "Life in Two Dimensions" (January 18, 1980), and a similar article, "Scientific Dreamers' Worldwide Cult," ran in Canada's *Maclean's* magazine (January 11, 1982). *Omni* (March 1983), in an article on "Flatland Redux," included a photograph of Dewdney shaking hands with an Astrian.

In 1984 Dewdney pulled it all together in a marvelous work, half nonfiction and half fantasy, titled *The Planiverse* and published by Poseidon Press, an imprint of Simon & Schuster. That same year he took over the mathematics column in *Scientific American*, shifting its emphasis to computer recreations. Several collections of his columns have been published by W. H. Freeman: *The Armchair Universe* (1987), *The Turing Omnibus* (1989), and *The Magic Machine* (1990).

An active branch of physics is now devoted to planar phenomena. It involves research on the properties of surfaces covered by a film one molecule thick, and a variety of two-dimensional electrostatic and electronic effects. Exploring possible flatlands also relates to a philosophical fad called "possible worlds." Extreme proponents of this movement actually argue that if a universe is logically possible—that is, free of logical contradictions—it is just as "real" as the universe in which we flourish.

In *Childhood's End* Arthur Clarke describes a giant planet where intense gravity has forced life to evolve almost flat forms with a vertical thickness of one centimeter.

The following letter from J. Richard Gott III, an astrophysicist at Princeton University, was published in *Scientific American* (October 1980):

> I was interested in Martin Gardner's article on the physics of Flatland, because for some years I have given the students in my general relativity class the problem of deriving the theory of general relativity for Flatland. The results are surprising. One does *not* obtain the Flatland analogue of Newtonian theory (masses with gravitational fields falling off like $1/r$) as the weak-field limit. General relativity in Flatland predicts no gravitational waves and no action at a distance. A planet in Flatland would produce no gravitational effects beyond its own radius. In our four-dimensional space-time the energy momentum tensor has 10 independent components, whereas the Riemann curvature tensor has 20 independent components. Thus it is possible to find solutions to the vacuum field equations $G_{\mu\nu} = 0$ (where all components of the energy momentum tensor are zero) that have a nonzero curvature. Black-hole solutions and the gravitational-field solution external to a planet are examples. This allows gravitational waves and action at a distance. Flatland has a three-dimensional space-time where the energy momentum tensor has six independent components and the Riemann curvature tensor also has only six independent components. In the vacuum where all components of the energy momentum tensor are zero all the components of the Riemann curvature tensor must also be zero. No action at a distance or gravity waves are allowed.
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> Electromagnetism in Flatland, on the other hand, behaves just as one would expect. The electromagnetic field tensor in four-dimensional space-time has six independent components that can be expressed as vector *E* and *B* fields with three components each. The electromagnetic field tensor in a three-dimensional space-time (Flatland) has three independent components: a vector *E* field with two compo-

nents and a scalar *B* field. Electromagnetic radiation exists, and charges have electric fields that fall off like $1/r$.

Two more letters, published in the same issue, follow. John S. Harris, of Brigham Young University's English Department, wrote:

As I examined Alexander Keewatin Dewdney's planiversal devices in Martin Gardner's article on science and technology in a two-dimensional universe, I was struck with the similarity of the mechanisms to the lockwork of the Mauser military pistol of 1895. This remarkable automatic pistol (which had many later variants) had no pivot pins or screws in its functional parts. Its entire operation was through sliding cam surfaces and two-dimensional sockets (called hinges by Dewdney). Indeed, the lockwork of a great many firearms, particularly those of the 19th century, follows essentially planiversal principles. For examples see the cutaway drawings in *Book of Pistols and Revolvers* by W. H. B. Smith.

Gardner suggests an exhibit of machines cut from cardboard, and that is exactly how the firearms genius John Browning worked. He would sketch the parts of a gun on paper or cardboard, cut out the individual parts with scissors (he often carried a small pair in his vest pocket), and then would say to his brother Ed, "Make me a part like this." Ed would ask, "How thick, John?" John would show a dimension with his thumb and forefinger, and Ed would measure the distance with calipers and make the part. The result is that virtually every part of the 100 or so Browning designs is essentially a two-dimensional shape with an added thickness.

This planiversality of Browning designs is the reason for the obsolescence of most of them. Dewdney says in his enthusiasm for the planiverse that "such devices are invariably space-saving." They are also expensive to manufacture. The Browning designs had to be manufactured by profiling machines: cam-following vertical milling machines. In cost of manufacture such designs cannot compete with designs that can be produced by automatic screw-cutting lathes, by broach-

> ing machines, by stamping, or by investment casting. Thus although the Browning designs have a marvelous aesthetic appeal, and although they function with delightful smoothness, they have nearly all gone out of production. They simply got too expensive to make.

Stefan Drobot, a mathematician at Ohio State University, had this to say:

> In Martin Gardner's article he and the authors he quotes seem to have overlooked the following aspect of a "planiverse": any communication by means of a wave process, acoustic or electromagnetic, would in such a universe be impossible. This is a consequence of the Huygens principle, which expresses a mathematical property of the (fundamental) solutions of the wave equation. More specifically, a sharp impulse-type signal (represented by a "delta function") originating from some point is propagated in a space of three spatial dimensions in a manner essentially different from that in which it is propagated in a space of two spatial dimensions. In three-dimensional space the signal is propagated as a sharp-edged spherical wave without any trail. This property makes it possible to communicate by a wave process because two signals following each other in a short time can be distinguished.
>
> In a space with two spatial dimensions, on the other hand, the fundamental solution of the wave equation represents a wave that, although it too has a sharp edge, has a trail of theoretically infinite length. An observer at a fixed distance from the source of the signal would perceive the oncoming front (sound, light, etc.) and then would keep perceiving it, although the intensity would decrease in time. This fact would make communication by any wave process impossible because it would not allow two signals following each other to be distinguished. More practically such communication would take much more time. This letter could not be read in the planiverse, although it is (almost) two-dimensional.

My linear checkers and chess prompted many interesting letters. Abe

Schwartz assured me that on the 11-cell checker field Black also wins if the game is give-away. I. Richard Lapidus suggested modifying linear chess by interchanging knight and rook (the game is a draw), by adding more cells, by adding pawns that capture by moving forward one space, or by combinations of the three modifications. If the board is long enough, he suggested duplicating the pieces—two knights, two rooks—and adding several pawns, allowing a pawn a two-cell start option as in standard chess. Peter Stampolis proposed sustituting for the knight two pieces called "kops" because they combine features of knight and bishop moves. One kop moves only on white cells, the other only on black.

Of course many other board games lend themselves to linear forms, for example, Reversi (also called Othello), or John Conway's Phutball, described in the two-volume *Winning Ways* written by Elwyn Berlekamp, Richard Guy, and John Conway.

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